A Technique for the Design of Multiplexers Having Contiguous Channels*

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Summary—**A general procedure for the design of multiplexers having contiguous channel frequency bands is presented. Using this procedure, the individual channel band-pass filters are designed from low-pass prototype filters having a resistive termination at one end only. The use of parallel-connected band-pass filters designed in this fashion, along with a susceptance-annulling network, is shown to be capable of giving a nearly constant input conductance across the operating band of a multiplexer. A three-channel design example using comb-line band-pass filters was worked out and its input admittance and attenuation characteristics were computed. This design was also constructed and tested. The computer and experimental results demonstrated the validity of the theory.**

Introduction

MULTIPLEXERS are often needed in order to split a single channel carrying many frequencies into a number of separate channels carrying narrower bands of frequencies. They are also often used for the inverse process of summing a number of channels carrying different bands of frequencies, so that all of the frequencies can be put in a single broad-band channel without the loss of energy which would otherwise occur due to leakage of energy from any one of the input channels into the other input channels.

It might at first appear that the design of multiplexers could easily be accomplished by simply designing several filters using any of various band-pass filter design procedures [1] and then connecting the filters in parallel or in series, as the case may be. However, useful as these filter design procedures are for multiplexer design, they must be accompanied by special techniques in order to avoid undesirable interaction between the filters, which could result in very poor performance.

A conceptually simple way of solving the multiplexer problem is to use directional filters [2]. Filters of this sort, which are all designed for the same terminating resistance, can be cascaded to form a multiplexer that in theory completely avoids interaction between filters. In many cases, it is a very practical way of dealing with multiplexing problems, even though it is by no means always the most practical way. Each filter will generally have some parasitic VSWR which will affect the system significantly, if many filters are to be cascaded. However, probably the greatest practical drawback of directional filters is that each resonator of each filter has two different orthogonal modes and, if more than one or two resonators are required per filter, the tuning of the filters may be difficult. Also, if multiplexers with contiguous pass-bands are to be designed, directional filters are not very suitable, since it would be difficult to give them the 3-db cross-overs usually desired. (For example, if at the desired crossover frequency the first directional filter removed half of the signal power, and if the second filter were also at a 3-db point, then the second filter would remove only half of the remaining power.)

Other design methods are available, if the channels of a multiplexer are quite narrow (say, of the order of 1 per cent bandwidth or less) and if the channels are separated by guard bands that are several times the passband width of the individual filters (or more). Then, relatively simple decoupling techniques should work quite well for preventing harmful interaction between filters [3]–[5].

When the channels of the multiplexer are contiguous, so that adjacent channels have attenuation characteristics that typically cross over at their 3-db points, decoupling techniques are not appropriate. In this case, the multiplexer should be designed as an integral unit. One design method—the method used in the synthesis of the multiplexer described in this paper—can be explained in terms of the circuit in Fig. 1. The schematic drawing represents an N-channel multiplexer consisting of specially designed band-pass filters. The filters are connected in parallel and a shunt, susceptance-annulling network is added at the input of the multiplexer to help provide a nearly constant total input admittance Y_{TN}. When properly designed, Y_{TN} will be nearly real and will approximate the generator conductance G_B across the operating band of the multiplexer. The band-pass filters that constitute the channels of the multiplexer are designed from a singly terminated, low-pass prototype filter rather than a doubly terminated prototype filter (*i.e.*, the prototype filter has a resistor termination at one end only). The reason is that, for the singly terminated prototype filter, the real part of the filter input admittance has the same frequency dependence as the filter transmission characteristic. This may be seen by observing the singly

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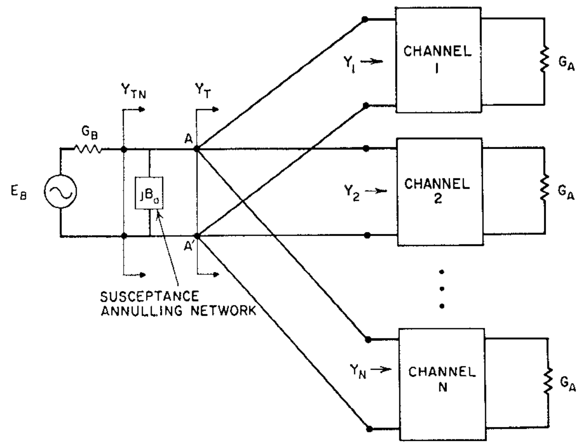

Fig. 1—A parallel-connected multiplexer.

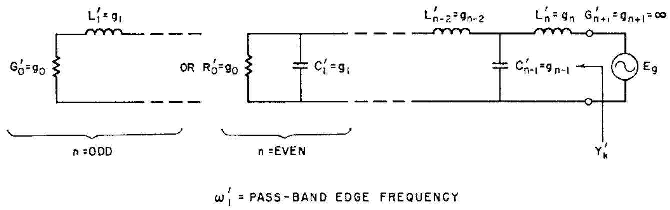

Fig. 2—Low-pass lumped element singly terminated prototype filter driven by a zero-impedance generator.

terminated low-pass prototype filter shown in Fig. 2.[1] This filter is driven by a zero-impedance generator at the right end and has a resistor termination at the left end. Because the filter is lossless, the power delivered to the load on the left in this circuit is given by

$$P = |E_g|^2 \text{ Re } Y_k' \tag{1}$$

where Y_k' is the admittance seen from the generator. Thus, if the filter has a Chebyshev transmission characteristic, for example, the Re Y_k' must also have a Chebyshev characteristic.

Fig. 3 shows typical Re Y_k' characteristics for Chebyshev, low-pass, prototype filters designed to be driven by zero-impedance generators. The input admittance Y_i of the channel filters in Fig. 1 are band-pass mappings of such low-pass, prototype, admittance characteristics. Notice that in Fig. 3 a low-pass prototype filter parameter g_{n+1}'' is defined. This parameter will be referred to in the discussion of the details of multiplexer design. However, it should be noted at this point that either g_{n+1}'' or its reciprocal corresponds to the geometric mean between the value of Re Y_k' at the top of the ripples and Re Y_k' at the bottom of the ripples. This admittance level for the low-pass prototype is analogous to the driving source admittance G_B in Fig. 1. That is, the multiplexer described in this paper is designed to have a Chebyshev Re Y_{TN} characteristic with G_B equal to the mean value of the ripples.

[1] Tabulated element values for such filters will be found in Matthaei [1] and Weinberg [6].

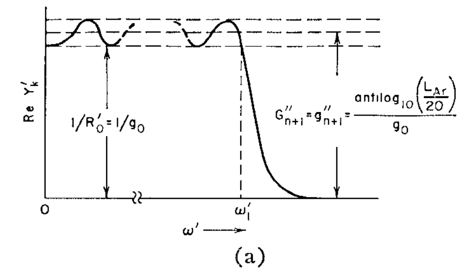

Fig. 3—Re Y'_k response of a low-pass prototype filter and definition of the quantity g''_{n+1}. (a) Case of n even. (b) Case of n odd.

Comb-Line Filter Description[2]

The multiplexer design techniques described in this paper are applicable for use with many different types of filter structures. However, to illustrate one application of these multiplexer design techniques, the use of comb-line filter structures will be assumed herein.

Fig. 4 shows a comb-line band-pass filter in strip-line form. The resonators in this type of filter consist of TEM-mode transmission-line elements that are short-circuited at one end and have a lumped capacitance C_k^s between the other end of each resonator line element and ground. In Fig. 4, lines 1 to n and their associated capacitances, C_1^s to C_n^s, constitute resonators, while lines 0 and $n+1$ are not resonators but simply part of impedance-transforming sections at the ends of the filter. Coupling between resonators is achieved in this type of filter by way of fringing fields between resonator lines. With the capacitors C_k^s present, the resonator lines will be less than $\lambda_0/4$ long at resonance (where λ_0 is the wavelength in the medium of propagation at midband), and the coupling between resonators is predominantly magnetic. Interestingly enough, if the capacitors C_k^s were not present, the resonator lines would be a full $\lambda_0/4$ long at resonance, *and the structure would have no pass-band* [8]. Without some kind of reactive loading at the ends of the resonator line elements, the magnetic and electric coupling effects would cancel each other, and the comb-line structure would become an all-stop structure.

For reasons described above, it is usually desirable to

[2] The design equations for comb-line filters and their derivation are given by Matthaei [7].

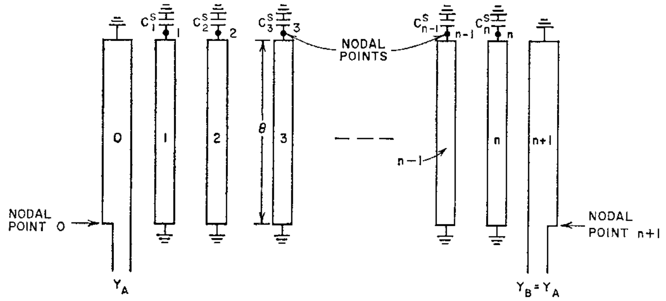

Fig. 4—A comb-line band-pass filter.

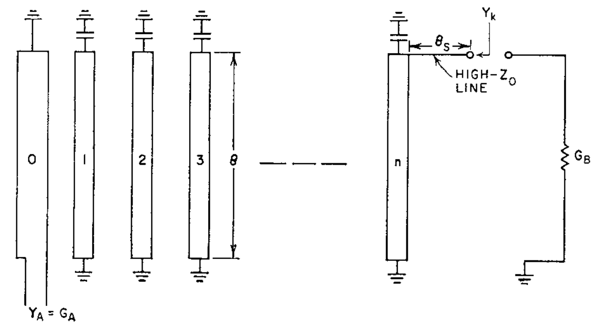

Fig. 5—A possible comb-line filter configuration for use in multiplexers.

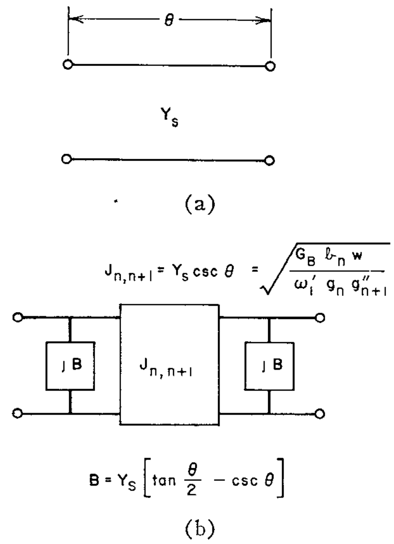

Fig. 6—Use of a high-impedance wire as a J-inverter.

make the capacitances C_k^s in this type of filter sufficiently large so that the resonator lines will be $\lambda_0/8$ or less long at resonance. Besides permitting efficient coupling between resonators (with sizeable spacings between adjacent resonator lines), the resulting filter will be relatively small. In this type of filter, the second pass-band occurs when the resonator line elements are somewhat over one-half wavelength long, so that, if the resonator lines are $\lambda_0/8$ long at the primary pass-band, the second pass-band will have its center at slightly over four times the frequency of the center of the first pass-band. If the resonator line elements are made to be less than $\lambda_0/8$ long at the primary pass-band, the second pass-band will be even further removed from the primary pass-band.

To adapt the comb-line filter design [7] for use in multiplexers, it was necessary to modify the input end of the filter in order to provide for coupling several filters to a common input junction. The modification used is shown schematically in Fig. 5. The comb-line filter in this figure is seen to include a fine-wire, high-impedance line at the input end.

Fig. 6 shows a high-impedance wire and its equivalent circuit when used as an admittance inverter $J_{n,n+1}$. (An ideal admittance inverter is defined as a device which operates like a quarter-wavelength line of characteristic admittance $J_{n,n+1}$ mhos at all frequencies [7].[3]) The The comb-line-filter design equations [7] of Matthaei, which utilize the admittance inverter concept, were modified to use this high-impedance-wire type of inverter at one end of the filter. The susceptance at one side of the inverter in Fig. 6 was compensated for by absorbing it into resonator n, while the susceptance B on the other side of the inverter was effectively absorbed into the susceptance-annulling network. In this case, the high-impedance wire type of coupling was suggested because it would help prevent the common junction from being crowded. However, it is desirable to keep the high-impedance wire short in order to avoid unwanted resonances, while not having it so small in diameter that it would increase the losses.

The design equations for the modified comb-line filter having a high impedance line in the input section are given in Getsinger [10]. These equations are equivalent to those of Matthaei [7] except for the input section. Hence, by use of the design data in Matthaei [7] along with the information in Fig. 6, the desired comb-line filter designs can be obtained.

A Computed Example Using Comb-Line Filters

Using the equations for the modified comb-line filter, a four-resonator, 10-per cent bandwidth filter design was worked out using a $n=4$, singly terminated, Chebyshev, prototype filter having 1-db ripple. The filter design was expressed analytically in an approximate form and its input admittance Y_k was computed using a digital computer. The results are shown in Fig. 7, normalized with respect to G_A. Note that Re Y_k/G_A is very nearly perfectly Chebyshev, while in the pass-band the slope of Im Y_k/G_A is negative on the average.

If band-pass filters with input admittance characteristics such as that in Fig. 7 are designed to cover contiguous bands, and if they are designed so that the Re Y_k/G_A characteristics of adjacent filters overlap at approximately their Re $Y_k/G_A = 0.5$ point[4] or slightly

[3] Cohn [9] describes impedance inverters. An admittance inverter is an admittance representation of an impedance inverter.

[4] By (1), it can be shown that the Re Y_k/G_A characteristic for a singly terminated Chebyshev filter can be computed using standard Chebyshev transfer functions. Equations for doing this are given in Getsinger [10].

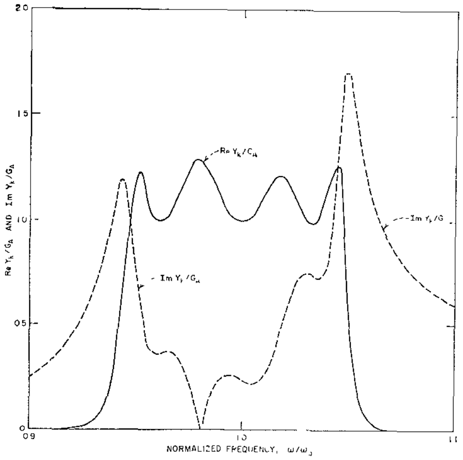

Fig. 7—Computed normalized Y_k of a modified comb-line filter.

below, then their total input admittance $Y_T = Y_1 + Y_2 + Y_3 + \cdots + Y_N$, when they are paralleled, will also have an approximately Chebyshev real-part characteristic. This was done in the example in Fig. 8. This figure shows the computed real-part characteristic for three, paralleled, comb-line filters with contiguous pass-bands, where the individual filters have input admittance characteristics as shown in Fig. 7 except for a shift in frequency. Fig. 9 shows the corresponding Im Y_T/G_A characteristic for this three-channel design.

Note that the Im Y_T/G_A characteristic in Fig. 9 is negative on the right side of the figure and that, on the average, the slope of the curve is negative throughout the operating band of the multiplexer. Since, by Foster's reactance theorem, the susceptance slope of a lossless network is always positive, the operating-band susceptance (Fig. 9) with its average negative slope can be largely cancelled by adding an appropriate lossless shunt branch (which will have a positive susceptance slope). In this case, a susceptance-annulling branch could consist of a short-circuited stub of such a length as to give resonance at the normalized frequency $\omega/\omega_0 = 1.02$, where the Im Y_T/G_A curve in Fig. 9 is approximately zero. Estimates indicate that, if the stub had a normalized characteristic admittance of $Y_s/G_A = 3.85$, the susceptance slope of the annulling network should be about right. Fig. 10 shows the normalized susceptance Im Y_{TN}/G_A after the susceptance-annulling network has been added. Note that although Im Y_{TN} has not been completely eliminated, it has been greatly reduced.

Since Re $Y_{TN}/G_A =$ Re Y_T/G_A, the total input admittance Y_{TN} with the annulling network in place is given by Figs. 8 and 10. To the extent that Y_{TN} approximates a constant conductance across the operating band, G_B and Y_{TN} in Fig. 8 will act as a resistive voltage divider, and the voltage developed across terminals A–A' of Fig. 1 will be constant with frequency. Under these conditions, the individual filters would have exactly the same response as if they were driven by zero-impedance generators (as they were designed to be

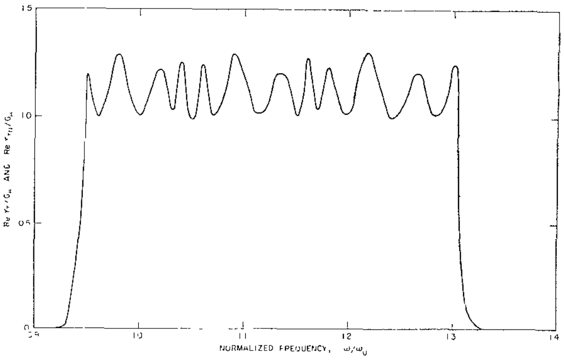

Fig. 8—Computed normalized Re Y_T of a 3-channel multiplexer using modified comb-line filters.

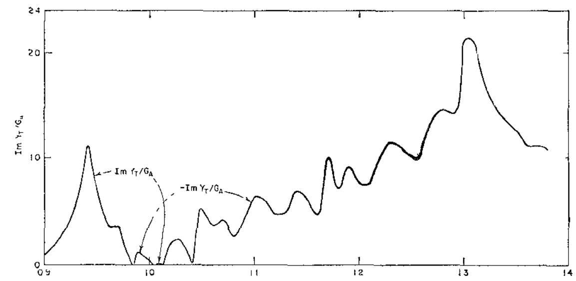

Fig. 9—Computed Im Y_T of a 3-channel multiplexer using modified comb-line filters.

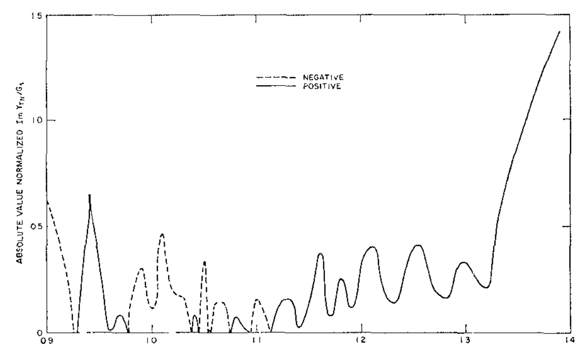

Fig. 10—Computed normalized Im Y_{TN} of a 3-channel multiplexer using modified comb-line filters and appropriate annulling network.

driven). However, since Y_{TN} only approximates a constant conductance, the performance will be altered somewhat from this idealized performance.

The driving generator conductance G_B for the trial multiplexer design was given a normalized value of $G_B/G_A = 1.15$, which makes the generator conductance equal to the mean value of the ripples in Fig. 8.[5] Fig. 11 shows the computed response of the multiplexer, while Fig. 12 shows the details of the pass-band response in enlarged scale. Note that the attenuation characteristics cross over at about the 3-db points, and that, although the filters were designed to have 1-db Chebyshev ripple when driven by a zero-impedance generator, the pass-

[5] This makes G_B correspond to $G_{n+1}'' = g_{n+1}''$ for the prototype as indicated in Fig. 3(a).

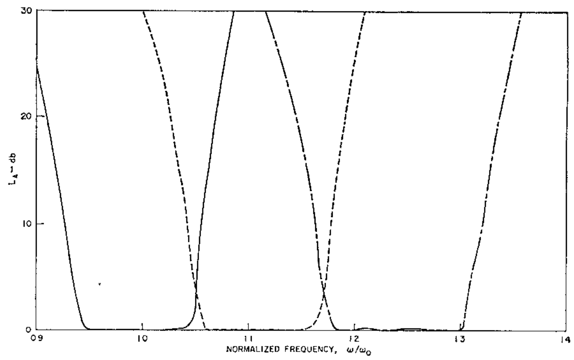

Fig. 11—Computed attenuation characteristics of the multiplexer channels.

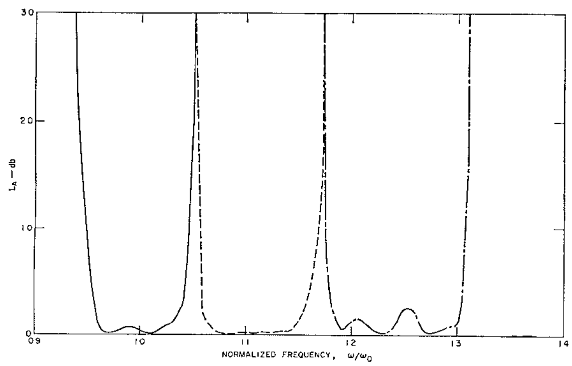

Fig. 12—Computed attenuation characteristics of the multiplexer channels in the pass-band.

band attenuation is much less than that in the completed multiplexer. This is largely due to the fact that adding the generator internal conductance G_B tends to mask out the variations in Re Y_{TN}. Also, choosing G_B to be equal to the mean value of Re Y_{TN} in the operating band tends to reduce the amount of mismatch that will occur.

Experimental Work

Using the design theory and equations previously discussed and referenced, a microwave multiplexer based on the preceding trial design was constructed. A sketch of the multiplexer is given in Fig. 13. The view in Fig. 13 is that seen when looking at the top of the multiplexer. The susceptance-annulling network of the multiplexer cannot be seen in this particular view. It consists of a specially constructed, low-impedance, coaxial line that is connected to the common junction and extends through the lower ground plane opposite the common input of the triplexer. The electrical length of the coaxial line was varied by sliding a short-circuit block.[6]

The multiplexer attenuation, after final tuning and adjusting of the annulling network for an optimum response, is given in Fig. 14. Fig. 15 gives the multiplexer attenuation in the pass-band using an expanded ordinate. The attenuation in the pass-band of the lower- and middle-frequency channels is typically 0.5 db. The

[6] For a more detailed description of the construction of the triplexer, see Matthaei [11].

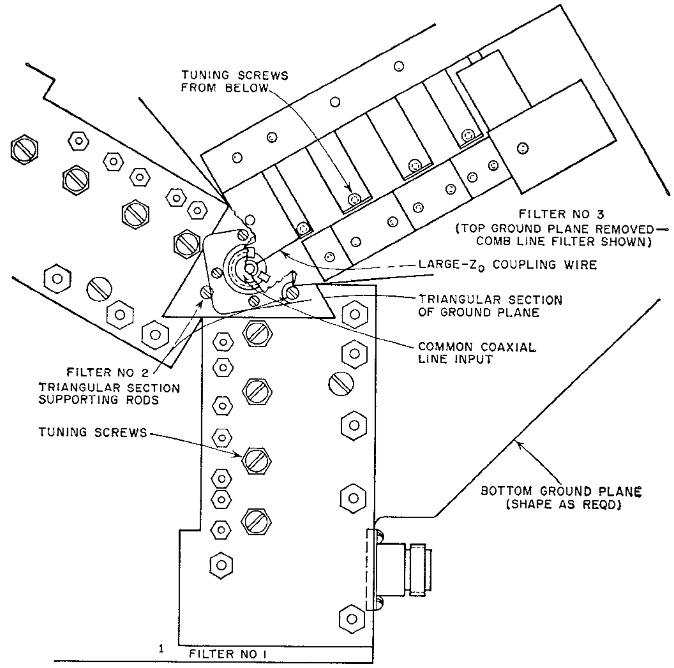

Fig. 13—Sketch of the top view of the 3-channel multiplexer showing one of the comb-line filters with its cover plate removed.

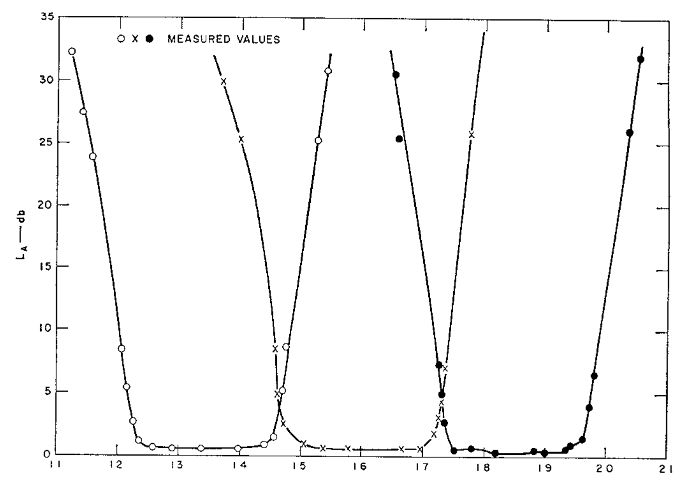

Fig. 14—Measured attenuation of the 3-channel multiplexer.

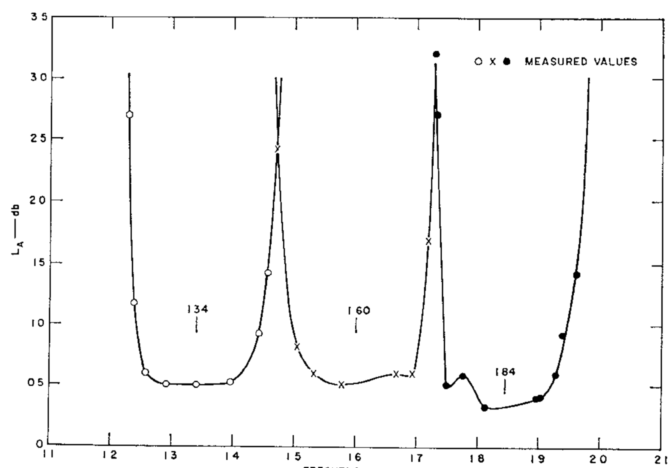

Fig. 15—Measured attenuation in the pass-band of the 3-channel multiplexer.

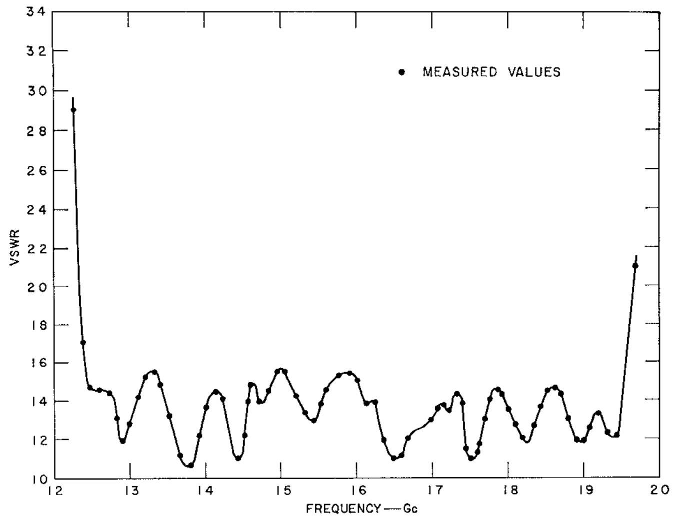

Fig. 16—Measured VSWR in the pass-band of the 3-channel multiplexer.

attenuation in the pass-band of the highest-frequency channel is approximately 0.4 db. The VSWR of the multiplexer (measured at the common input) is shown in Fig. 16 and is less than 1.6 throughout the pass-band except at the multiplexer band edges.

The 3-db percentage bandwidths of each channel were determined. These were defined as the difference of the frequencies at which the attenuation is 3 db, divided by the arithmetic mean of those frequencies. The bandwidths were found to be 17, 16, and 13 per cent for the lowest-, middle-, and highest-frequency channels, respectively. These percentages are larger than the corresponding values for the triplexer example previously discussed, whose channel 3-db percentage bandwidths were 11 per cent each. "Bandwidth spreading" has been noted in comb-line filters before [7]. However, in the case of a previous experimental comb-line filter having a design bandwidth of 10 per cent [7], the bandwidth spreading was less than in the present cases. Bandwidth spreading is believed to be due to coupling effects beyond nearest neighbor line elements, which were neglected in the derivation of the design equations for comb-line filters. It can be compensated for by increasing the spacings between the resonators slightly.

Conclusions

A design theory for multiplexers having contiguous channels was presented. The examples discussed were cases of multiplexers using filters connected in parallel. However, the same approach can be carried out on a dual basis for multiplexers with the filters connected in series. (In the series case, a series reactance-annulling network is required instead of a shunt, susceptance-annulling network.) The computed performance of the trial design showed that the proposed technique works very well. The measured performance of the triplexer based on the design compared well with the computed performance.

One of the main difficulties of this approach for some applications is that the filters are assumed to be interconnected at a point. Space considerations may make it difficult to interconnect a very large number of filters, although in many cases three or four can be interconnected very easily. The series-inductance coupling proposed for the case of multiplexing comb-line filters provides one way of interconnecting the filters while keeping them separated from each other as much as possible.

Acknowledgment

The experimental three-channel multiplexer was constructed by R. Pierce and C. A. Knight. P. R. Rezneck took the experimental data. The computed example of the three-channel multiplexer design was programmed for the Burroughs 220 computer by J. R. Herndon.

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